

## **Johann Carl Friedrich Gauss (1777-1855)**

Carl Friedrich Gauss, the "Prince of Mathematics," exhibited his calculative powers when he corrected his father's arithmetic before the age of three. His revolutionary nature was demonstrated at age twelve, when he began questioning the axioms of Euclid. His genius was confirmed at the age of nineteen when he proved that the regular  $n$ -gon was constructible, for odd  $n$ , if and only if  $n$  is the product of distinct prime Fermat numbers. (Click to see construction of regular 17-gon.) At age 24 he published *Disquisitiones Arithmeticae*, probably the greatest book of pure mathematics ever.

Although he published fewer papers than some other great mathematicians, Gauss may be the greatest theorem prover ever. Several important theorems and lemmas bear his name; he was first to produce a complete proof of Euclid's Fundamental Theorem of Arithmetic (that every natural number has a unique expression as product of primes); and first to produce a rigorous proof of the Fundamental Theorem of Algebra (that an  $n$ -th degree polynomial has  $n$  complex roots). Gauss himself used "Fundamental Theorem" to refer to Euler's Law of Quadratic Reciprocity; Gauss was first to provide a proof for this, and provided eight distinct proofs for it over the years. Gauss proved the  $n=3$  case of Fermat's Last Theorem for a class of complex integers; though more general, the proof was simpler than the real integer proof, a discovery which revolutionized algebra. Other work by Gauss led to fundamental theorems in statistics, vector analysis, function theory, and generalizations of the Fundamental Theorem of Calculus.

Gauss built the theory of complex numbers into its modern form, including the notion of "monogenic" functions which are now ubiquitous in mathematical physics. Gauss was the premier number theoretician of all time. Other contributions of Gauss include hypergeometric series, foundations of statistics, and differential geometry. He also did important work in geometry, providing an improved solution to Apollonius' famous problem of tangent circles, stating and proving the Fundamental Theorem of Normal Axonometry, and solving astronomical problems related to comet orbits and navigation by the stars. (The first asteroid was discovered when Gauss was a young man; he famously constructed an 8th-degree polynomial equation to predict its orbit.) Gauss also did important work in several areas of physics, and invented the heliotrope.

Much of Gauss's work wasn't published: unbeknownst to his colleagues it was Gauss who first discovered non-Euclidean geometry (even anticipating Einstein by suggesting physical space might not be Euclidean), doubly periodic elliptic functions, a prime distribution formula, quaternions, foundations of topology, the Law of Least Squares, Dirichlet's class number formula, the key Bonnet's Theorem of differential geometry (now usually called Gauss-Bonnet Theorem), the butterfly procedure for rapid calculation of Fourier series, and even the rudiments of knot theory. Also in this category is the Fundamental Theorem of Functions of a Complex Variable (that the line-integral over a closed curve of a monogenic function is zero): he proved this first but let Cauchy take the credit. Gauss is widely agreed to be the most brilliant and productive mathematician who ever lived and many would rank him #1; however several of the others on the list had

more historical importance. Abel hints at a reason for this: "[Gauss] is like the fox, who effaces his tracks in the sand."

Gauss once wrote "It is not knowledge, but the act of learning, ... which grants the greatest enjoyment. When I have clarified and exhausted a subject, then I turn away from it, in order to go into darkness again ..."